Abstract

Proof lies at the heart of mathematics though research shows that proof is an elusive concept for many mathematics students. This descriptive-assessment study measured the proof-writing performance of students vis-à-vis Van Hiele's Model of Geometric Proof level. Data disclosed that education students of Southern Leyte State University performed “below average” in proving geometrical proposition. At the conclusion that students stuck at the analysis level, it was recommended that proving should be an integral part in any classroom discussion to enhance students’ particular skills on recognition, analysis, order, deduction and rigor.

Keywords: visualization, analysis, order, deduction, rigor, writing proof, proving performance, Van Hiele

1.0 Introduction

For decades, proof has been given increased attention in many countries around the world (Knipping, 2004; Cirillo & Herbst, 2011). This is primarily because proof is considered the basis of mathematical understanding and is essential for developing, establishing, and communicating mathematical knowledge (Stylianides, 2007). More specifically, Wu (1996) argued that anyone who wants to know what mathematics is about, must learn how to write, or at least understand, a proof. This study claims that education students write proof at the analysis level of Van Heile’s model in geometrical proving.

It has been established that proving is a skill necessary for students to be successful and efficient in the study of Geometry. In fact, Hanna (2000) considered proving as the most important tool in Mathematics (Dimakos et al., 2007) and without it, Mathematics is not essentially Mathematics. After all, proof is probably the most important tool used in geometry. Corollary to this, the National Council of Teachers of Mathematics (NCTM) recommended reasoning and proof as fundamental aspects of mathematics (NCTM, 2000). It was then that writing proof has been emphasized as a necessary component in the university mathematics curriculum. Proof had
become a part of every school mathematics experiences (Knuth, 2005) and a natural, ongoing part of classroom discussions (Kennedy, 2002).

Despite these attempts, a number of studies continued to provide evidences of students’ weaknesses in writing formal proofs. A research conducted by Senk (1985) provided evidence that students, even after having participated in an introductory teaching of proof, could neither realize the necessity of deductive proofs nor distinguish the various kinds of mathematical reasoning. The study of Dimakos (2007) found that students have great difficulty in writing formal proofs in Geometry. He added that although teaching students to write proofs has been an important goal of the geometry curricula on an international level, contemporary students still rank doing proofs in geometry among the least important, most disliked and most difficult of school mathematics topics. A number of students don’t know what a proof constitutes (Godino & Recio, 2001), and they are even unable to confirm the validity of a proof (Selden & Selden, 2003). According to the study of Healy & Hoyles, (1998), students found it easier to evaluate proofs rather than to construct and formulate ones.

As part of the mathematics curriculum in a university, proving is essentially a very important aspect in the study of geometry. This study, therefore, aimed to evaluate the learning performance and learning level of education students in Southern Leyte State University-Tomas Oppus in proving geometrical proposition vis-à-vis Van Heiles’ Model of Geometric Thought. This will inform mathematics educators on the status of education students’ proof-writing ability and will encourage them to devise strategies to help students achieve the highest level of geometric thought.

2.0 Conceptual / Theoretical Framework

This study is theoretically anchored on Van Heile’s Model of the Development of the Geometric Thought. The model consists of five levels of understanding concepts through which a student move sequentially to geometric proving (Usiskin, 1980). He developed a procedural activity which categorizes students learning level — visualization, analysis, order, deduction and rigor. Assisted by appropriate instructional experiences, the model asserts that the learner moves sequentially from the initial or basic level (visualization) through the highest level (rigor), which is concerned with formal abstract aspects of deduction. Few students are exposed to the latter level (Crowley, 1987). Figure 1 shows this model.
Level 1: Visualization. At this initial stage, students are aware of space only as something that exists around them. Geometric concepts are viewed as total entities rather than as having components or attributes.

Level 2: Analysis. At this level, an analysis of geometric concepts begins. Through the use of observation and experimentation students begin to discern the characteristics of figures. These emerging properties are then used to conceptualize classes of shapes. Thus, figures are recognized as having parts and are recognized by their parts.

Level 3: Order. At this level, students can establish the interrelationships of properties both within figures and among figures. Thus, they can deduce properties of a figure and recognize classes of figures. Class inclusion is understood and definitions are meaningful. Informal arguments can be followed and given. The student at this level, however, does not comprehend the significance of deduction as a whole or the role of axioms. Empirically obtained results are often used in conjunction with deduction techniques. Formal proofs can be followed, but students do not see how the logical order could be altered nor do they see how to construct a proof starting from rent or unfamiliar premises.

Level 4: Deduction. At this level, the significance of deduction as a way of establishing a theory within an axiomatic system is understood. The interrelationship and role of undefined terms, axioms, postulates, definition, theorems, and a proof is seen. A person at this level can construct, not just memorize, proofs; the possibility of developing a proof in more than one way is seen; the interaction of necessary and sufficient conditions is understood; distinctions between a statement and its converse can be made.

Level 5: Rigor. At this stage
the learner can work in a variety of axiomatic systems, that is, non-Euclidean geometries can be studied, and different systems can be compared. Geometry is seen in the abstract.

3.0 Research Design and Methods

A total of thirty education students enrolled in the subject geometry in Southern Leyte State University-Tomas Oppus during School Year 2008-2009 participated in this descriptive study. The tools utilized in this study were the Geometry Proof Test adapted from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG), a revised Van Hiele level of Geometrical Learning Questionnaire, and a Researcher Made Questionnaire.

Geometry Proof Test. These questions were adopted from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) test that was designed to arrive at a measure of proof-writing achievement. These were non-overlapping tests consisting only of six items: Fill-in proof; Conditional Statement; Synthetic Proof without hints; Analytic Proof without hints; Local Deduction; and Synthetic Proof with hints. This test tried to introduce students to different proving types to measure their level of knowledge and their ability to prove different proving styles. The scoring of the proof writing test was also adapted so as to evaluate properly the responses of the students.

Geometrical Learning Questionnaire.
An instrument from Pierre Marie Van Hieles’ level theory which aimed to measure the respondents’ level of learning based on their responses. It is a set of behaviors that identify the level of the students’ learning in geometrical proving. This was used to obtain the perceived learners’ level of geometrical learning after they worked on the Geometrical Proof Test.

4.0 Results and Discussion

Performance in Proving Geometrical Proposition

Education Students of Southern Leyte State University – Tomas Oppus performed very poor in proving geometrical propositions. This shows the low performance of students in doing formal proofs in geometry as stated by previous researches. Evidence presented in this study demonstrates that students’ mathematical and geometric reasoning seems to be contributing to their lack of achievement in proof. Per evaluation of the students’ paper, this performance is due to students’ poor organization of reasoning ability and failure to see relationships of figures. Mansi (2003) stated that the lack of student achievement in proof has becoming increasingly apparent in this modern era.

A deeper view on the result shows that education students found
Table 1. The performance profile of SLSU – TO education students in proving geometrical proposition

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Average</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above Average</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1</td>
<td>10.23</td>
<td>Below Average</td>
</tr>
<tr>
<td>Below Average</td>
<td>26</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Learning Level in Geometrical Proving as Reflected in Students’ Solution Papers

Recognition. In this study, a high percentage of education students reached the recognition level. Students achieved this high percentage because at this level they do not make decisions based on reasoning but on perception (Mason, 2008). Generally, students were able to recognize figures and geometrical concepts which show that they were able to identify shapes into categories on the basis of how the shapes are alike (Shaughnessy & Burger, 1986).

It is, however, observed that not all students were able to reach recognition level. This suggests that there were students who failed to illustrate and identify figures stated in the conditional statement. This recognition symptom would lead to students’ difficulty to perform accurate calculations and analysis. This indicates that students needed to develop their level of recognition on the figures since thinking visually was an important dimension of intellectual activity that helped them to find a solution of a problem (Conroy et al., 2005).

Analysis. In this level, it was observed that the students started to encounter the difficulty in establishing properties of shapes. Finding shows that respondents reached analysis level and implied that analysis was evident in a relative number of students. Such evidence was good in comparison to the study of Senk (1989) as quoted by Clements & Battista (1992) who said that less than twenty-two
percent of students were able to reach this level. This could be inferred that only few students saw figures as collections of properties. They looked at figures as a whole as manifested in the recognition level but was unsuccessful to recognize the properties and features behind the figure (Clements & Battista, 1992). Thus students were barred towards further understanding because they failed to understand the language that the figure wanted to communicate (Lee, 2006).

**Order.** In this stage, a sudden and observable decrease of frequency of the number of students who achieved this level was very apparent. The result indicates that students failed to create meaningful relationships between the properties of figures. This further indicates that students were unsuccessful to extend their examination on the properties of the figures by recognizing the need for axioms, postulates and theorems that clarified a system of properties (Malloy, 2002). This suggested that as the learning level went higher, the lesser the number of students was there to achieve it. This could be understood that only few students were able to master the previous level that caused them not to achieve the next level (Mason, 2008). This happened because the levels were not situated in the subject matter but in the thought of man (Dindyal, 1999). Thus, non-mastery of the previous level meant failure to achieve the next level of learning.

**Deduction.** Results showed that very few respondents reached this level. This suggests that only few were able to start constructing a sequence of statements that logically justified a conclusion. Furthermore, it explained that a bigger number of students were not able to understand the role and application of the axiomatic system in proving geometrical propositions. Crowley said as quoted by Malloy (2002) that many students were still unable to comprehend the significance of proof, and thus, failed to complete it. Even several studies reported that the deduction ability among students who have studied school geometry was nearly absent (Usiskin, 1980).

Nevertheless, as reflected in

![Figure 1. Learning level in geometrical proving as reflected in students’ solution papers.](image-url)
Rigor. In general, the result shows that students hardly got into this level. Only few students were able to operate on the abstract level of geometry. This simply explained Dimakos (2007) who said that writing proofs, even in Euclidean Geometry, was difficult for students. This expressed the student’s weak verifications on analysis. It further indicated that students failed to prove that the parts of the figure are congruent which consequently led to an unsuccessful proving of the figure as a whole. This assertion held true in comparison with the study conducted by Healy and Hoyles as quoted by Clements and Battista (1992). Their research project on “Understanding the Proof” using the CDASSG instrument held in America confirmed that writing proof was difficult for most students.

Information reflects a decreasing trend in the attainment of learning levels in geometrical proving. This is consistent with the study of Senk (1985) and Usiskin (1980) who showed that only few students reached the highest level of Van Heile’s Model. It was evident in the overall result that students were only good in looking at the figure as a whole but failed to see that figures were expressions of a collection of properties. Students were good at the syncretic or visual level but were very weak at descriptive (verbal) and abstract (relational thinking) level (Clements and Battista, 1992). Despite this result, the papers of the students indicated a very strong desire to work on the proving processes in their own pace of understanding the system of operation in proving. Such motivation indicates a strong will to construct a valid proof on geometrical propositions though, ultimately, they failed to make it.

The result of the study, however, surpasses the expectation of the researcher. There were education students who reached the vigor level in their effort to construct proof in geometry. Students who reach this stage are those with strong determination to write a complete valid proof in geometry. They are those whose desire and willingness to construct valid proofs complements their reasoning ability enabling them to work various axiomatic systems. Interest and desire pushes education students to overcome difficulties in writing proof and opens the way to a greater possibility of writing more valid proofs in geometry.

The Learning Level in Geometrical Proving as Perceived by the Students

Individual description revealed students’ self-assessment on the different behaviors required for students to perform successfully in each level. Respondents claimed that they employed the various behavioral practices specified for each level. Briefly, they claimed that they reach the highest level of Van Heile’s Model of Geometric Thought. Their responses, however disclosed the fact that the necessary behaviors for proof-construction activity were not always employed and exercised. Evident to this is the
The frequency of students’ exposure in writing proof strengthens their capacity and confidence to complete a valid proof skill as they engaged in the process of argumentation (Ball & Bass, 2003; Diezmann, Watters, & English, 2002). Ultimately, this signified that students’ exposure and constant experience of proving greatly affect their behaviors towards proving activity. As mentioned by Dindyal (1999), progression from one level to the next was not the result of maturation or natural development. It relied on the quality of the experience that one was exposed to.

Table 2. Learning level in geometrical proving as perceived by the students.

<table>
<thead>
<tr>
<th>Learning Level</th>
<th>Behavioral Indicators</th>
<th>Modal Response</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognition</td>
<td>• Identify and name figures</td>
<td>Sometimes</td>
<td>Sometimes</td>
</tr>
<tr>
<td></td>
<td>• Recognizes shape from the given statement</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td>Analysis</td>
<td>• Identify the properties of the figure</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Measure and observe the properties</td>
<td>Often</td>
<td>Sometimes</td>
</tr>
<tr>
<td></td>
<td>• Find resemblance in the picture</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td>Order</td>
<td>• Manipulate relationships of geometric patterns</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Properties are ordered and arranged</td>
<td>Seldom</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Use informal, deductive language</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Doing solutions of geometric measures</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Making hypothesis</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Testing hypothesis</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td>Deduction</td>
<td>• Constructs proof using postulates or definitions</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Manipulate intrinsic relationship among figures</td>
<td>Sometimes</td>
<td>Sometimes</td>
</tr>
<tr>
<td></td>
<td>• Applies logical thinking</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Distinguish proposition and its converse</td>
<td>Seldom</td>
<td></td>
</tr>
<tr>
<td>Rigor</td>
<td>• Work on different geometrical systems</td>
<td>Sometimes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Check with logical thinking</td>
<td>Seldom</td>
<td>Sometimes</td>
</tr>
<tr>
<td></td>
<td>• Verify reasoning</td>
<td>Sometimes</td>
<td></td>
</tr>
</tbody>
</table>

modal responses that they manifest as reflected in Table 2. They failed to practice the behaviors “always”; instead, they have it executed “sometimes”. Viewed from the behavioral perspective, students’ poor performance in writing geometrical proof is attributed to their perceived lack of exercise and practice in constructing valid proofs - an activity necessary to develop and to improve students’ proving skill. If done always, this perceived opportunity could have improved their competence in writing valid proofs as classroom opportunity could develop students’ reasoning skill as they engaged in the process of argumentation (Ball & Bass, 2003; Diezmann, Watters, & English, 2002). Ultimately, this signified that students’ exposure and constant experience of proving greatly affect their behaviors towards proving activity. As mentioned by Dindyal (1999), progression from one level to the next was not the result of maturation or natural development. It relied on the quality of the experience that one was exposed to.
in geometry. If students are seldom exposed to a certain task, they really would have the difficulty in completing the task. This is similar in writing proof. Knowledge is cultured. No matter how intelligent student is, if student are not exposed to the task, the latter will not be completed. Exposure begets creativity in finding ways to construct valid proof in geometry.

5.0 Conclusion

Though all students claimed “perceptually” that they reached the highest level of Van Heile’s Model of Geometrical Thought, actual performance shows that majority of them were only at the Analysis Level. This study concludes that in a proof-writing activity, perceived learning performance is not necessarily the same as the actual performance.

Thus, it is recommended that proving and reasoning must be an integral part in any classroom discussion to enhance students’ particular skills on recognition, analysis, order, deduction and rigor. This means that students and teachers should give more time to understand the language of Mathematics through constant exposure in classroom proving activities and discussion to correctly reflect their understanding on their papers. This would lead students to develop their understanding on the relationship between plausible and demonstrative reasoning which will help their consistency on what they believe and what they actually have written. This would also avoid inconsistency of students’ belief of their performance and their actual performance in geometrical proving.

6.0 References Cited


